

# The Contribution of Air to Ultrasonic Friction Reduction

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**Abstract**—The origin of friction reduction on an ultrasonically vibrating plate has been the subject of debate. Recent work suggests that friction may be reduced due to intermittent contact caused by bouncing upon the vibrating surface [8], leaving the question of whether other phenomena such as levitation on a squeeze film of air also play a role. To probe the contribution of squeeze film levitation, we investigated the dependence of the friction reduction effect upon air pressure. An artificial finger was placed inside a vacuum chamber, touching an ultrasonic friction reduction device composed of a glass plate vibrated by piezo-actuators. Friction between the finger and the glass was measured by rotating the finger with a motor, and measuring the motor's torque load. Decreased friction is signaled by decreased motor current. Compared to atmospheric pressure, a 98% vacuum inside the chamber was observed to markedly diminish the friction reduction effect, suggesting that squeeze film levitation does indeed play a substantial role in ultrasonic friction reduction.

## I. INTRODUCTION

Touchscreens are replacing mechanical interfaces, such as keyboards and mice, as the primary means of interaction with smart devices. The replacement offers advantages to developers who can now display and update an interface at will. Examples include virtual keyboards as well as virtual book pages and all manner of sliding widgets. However, despite the name and underlying physicality of many UI metaphors, touchscreens offer little in actual touch sensation: while they can display almost any visual or auditory stimulus, every interaction feels like a smooth flat screen. What if a touchscreen could actually feel like the rough paper of a book page, or like the buttons and divots of a keyboard? The realism that such haptic feedback could provide, as well as the as-yet undiscovered possibilities in our interaction with virtual displays, have motivated development of surface haptic displays that provide programmable touch sensations.

One way to provide touch feedback on screens is via ultrasonic friction modulation. By exciting a plate with transverse ultrasonic vibrations, the friction that a finger experiences on the plate can be dramatically reduced [1]. Modulating the amplitude of vibration, and consequently the friction, enables a wide range of virtual textures and surface features to be displayed [2]. Although the technique is known to work, it would be valuable to have a deeper understanding of this effect in order to build more efficient and effective devices.

### A. Squeeze Film Theory

For some time, the leading explanation for ultrasonic friction reduction postulated that the surface is lubricated by a thin film of air. Lubricating air bearings can be generated straightforwardly by supplying extra air to an interface, but in the 1960s researchers began exploring methods to pressurize the gap between two surfaces via the squeeze film effect [3], [4]. In 1964, Erik Salbu of IBM demonstrated that experimental measurements matched mathematical predictions of squeeze film force produced by a vibrating plate, provided the air gap above the vibrating surface is small enough and vibration fast enough [4]. These conditions are described by the non-dimensional squeeze number

$$\sigma = \frac{12\eta L^2 \omega}{p_{atm} u^2} \quad (1)$$

where  $L$  is the length of the plate,  $\eta$  is air viscosity,  $\omega$  the excitation frequency,  $p_{atm}$  the atmospheric pressure, and  $u$  the gap distance between the plate and the flat surface above it. At high enough squeeze numbers, due to a very small gap or a very large frequency, a squeeze film of air develops. At these large squeeze numbers, the air cannot easily escape the narrow sides of the gap, and instead stays trapped between the two surfaces. At this point, viscous forces of air being pumped in and out of the gap are overtaken by elastic forces of the trapped air, causing it to act like a non-linear spring [5]. This creates an over-pressurization when averaged over time, levitating the top surface and creating the air bearing effect [4], [6].

A common operating frequency for an ultrasonically vibrating screen is in the range of  $\omega \approx 30$  kHz. The air gap between a finger and the screen, which could initially exist around the small asperity tips the finger actually rests upon, would be on the order of a few microns. These values result in a squeeze number several orders of magnitude larger than the threshold value  $\sigma = 10$  above which a squeeze film could be expected to develop.

At these characteristic length scales, the squeeze pressure that produces a levitation force is governed by the Reynolds lubrication equation:

$$\frac{\partial}{\partial x} \left( \frac{\bar{u}^3 p}{12\mu} \frac{\partial p}{\partial x} \right) = \frac{\partial (p\bar{u})}{\partial t} \quad (2)$$

where  $\bar{u}$ ,  $p$  are the instantaneous gap size and pressure. However, for squeeze numbers above 150, little air escapes the gap [6], and the squeeze film pressure can be approximated with Boyle's Law, resulting in a time-averaged pressure

$$P_{squeeze} = \frac{5}{4} \frac{a^2}{u^2} P_{atm} \quad (3)$$

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where  $a$  is the vibration amplitude of the plate and  $p_{atm}$  is atmospheric pressure. This equation has been obtained multiple times; see [7], [4] for the time-averaged solution or [8] for a description of setting up a time-varying dynamical model.

### B. Squeeze Film Applied to Fingertips

Squeeze film theory has been extensively developed and proven for flat metal surfaces, where the levitating surface is either held in place or not expected to experience much displacement in relation to the vibrating surface [6]. Watanabe et al. introduced the idea of ultrasonically vibrating surfaces for tactile friction displays in the mid 1990s, postulating that the 'air smoothness' feeling was a result of the squeeze film effect acting on human fingers [1]. Since then, multiple research groups have cited squeeze films as the cause of friction reduction in tactile displays [9], [2]. Recent attempts to quantitatively relate squeeze film force to friction levels have had mixed results; Watanabe et al. [1], and later Sednaoui et al. [10] reported that their squeeze film model failed to predict friction dependence on increasing vibration amplitudes, raising the question of whether the present understanding of squeeze film can fully explain the mechanisms of friction reduction.

Recent work by Wiertlewski et al. [7] suggests that a squeeze film model can indeed predict friction levels over a wide range of vibration amplitudes when considering load sharing between squeeze film pressure and skin surface asperities. The additional pressure provided by a squeeze film decreases friction by reducing the number of asperities of the skin that are in intimate contact with the surface. Because the squeeze film pressure is inversely proportional to the square of the average gap and at the same time is responsible for increasing this gap, the skin settles onto a time-averaged levitation distance that corresponds to the balance between the external load applied by the finger, the load supported by the skin surface, and the squeeze film pressure:

$$P_{external} = P_{contact} + P_{squeeze} \quad (4)$$

which translates to the following force balance equation when  $P_{contact}$  is modeled by a multi-scale contact law [11]:

$$P_{external} \left( 1 - \exp\left(\frac{-u + u_0}{u_{rms}}\right) \right) = \frac{5}{4} \frac{a^2}{u^2} P_{atm} \quad (5)$$

where  $u_0$  is nominal gap size and  $u_{rms}$  is the root mean square of the asperity height profile, i.e. the roughness of the finger surface. This equation shows that the gap  $u$  is non-trivial and a non-linear function of the vibration. Friction modulation is then found using

$$\mu/\mu_0 = \exp\left(\frac{-u + u_0}{u_{rms}}\right) \approx \exp\left(-a^2 \frac{5 P_{atm}}{4 u_0^2 P_{external}}\right) \quad (6)$$

A full derivation of this approximation is in [7]. As a consequence, the relationship between friction modulation and the amplitude of vibration is also non-trivial and non-linear, but can be approximated with a Gaussian function. The spread of this Gaussian approximation depends inversely

on atmospheric pressure, meaning that under a greater atmospheric pressure the same reduction in friction is attained at a lower vibration amplitude.

### C. Alternative to Squeeze Film Theory

Another explanation for vibration-induced friction reduction is intermittent contact. As the vibrating surface moves up and down, real contact with the finger may occur for only a small portion of each vibration cycle. Several recent experiments highlight the fact that the finger is indeed actively bouncing upon the vibrating surface during friction reduction. Tracking the surface of a human finger in relation to a vibrating plate with a Laser Doppler Vibrometer reveals that the finger surface moves out of phase and on the same order of height magnitude as the plate itself, which is characteristic of bouncing [8], [12].

Artificial fingers that are built to resemble human fingers in terms of softness and shape also show friction reduction, and also move as if bouncing. We can also build artificial fingers that experience very different amounts of friction reduction, depending upon the damping properties of the skin material [13]. Artificial fingers that experience little to no friction reduction are typically those with less damping, and their surface motion appears to be more in phase with the vibrating plate [8]. This presents an intuitive explanation for why finger material construction could play such a large role in frictional behavior: a finger resting on a vibrating plate is a dynamical system which can be expected to behave differently with different damping or stiffness parameters. Changing those parameters changes how the finger bounces or fails to bounce upon the plate surface, thereby affecting the average distance between the finger and plate.

If intermittent contact is alone responsible for friction reduction, the implications could be significant for other types of haptic devices. For example, moments of impact could be exploited to push the finger sideways, creating active lateral forces in addition to friction modulation. This possibility was explored in [12], but the limited strength of forces achieved left open the question of whether the effect could be strengthened or if it is ultimately limited by a lubricating squeeze film of air. Additionally, Wiertlewski et al. argue that moments of impact do not appear in their data [7]. When illuminating the finger contact patch stroboscopically while touching a vibrating plate, they observed no flash of brightness that should accompany the impact of a bouncing finger slamming into the plate surface without a cushion of air present to soften the impact.

### D. Squeeze Film at Low Pressure

The open question of whether a squeeze film exists, and if so how large a role it plays in friction reduction, continues to linger. A squeeze film could be the main mechanism of friction reduction, or it could work in conjunction with intermittent contact, or it could be entirely unimportant. A straightforward approach to testing this theory is to remove the air and observe what effect that has on friction reduction.

Indeed, Ben Messaoud et al. recently performed this experiment with a live human finger in a low pressure chamber [14]. At 0.5 atmosphere pressure they found ultrasonic friction reduction lessened by about 20%. Pressures lower than 0.5 atmosphere were not tried due to safety concerns.

In this paper, we add to the evidence that a squeeze film does indeed contribute to ultrasonic friction reduction. Instead of a human finger, we use an artificial finger constructed to deform and experience friction reduction similarly. This allows us to reduce ambient pressure dramatically, while eliminating some of the troublesome features of real tissue, such as variable amounts of sweating. We observe changes in friction between the artificial finger and a vibrating plate from atmospheric pressure (1 atm) down to 0.02 atm, and compare these results to expected changes based on the model described in equations 5 and 6.

## II. MATERIALS AND METHODS

### A. Artificial Finger

The artificial finger used in these experiments was constructed to mimic the biological layers of a real human finger. It consists of a rubber-like 0.5mm thick 3D printed skin shell, surrounding a soft foam layer and an inner aluminum bone core. The surface of the finger was coated in a thin layer of white acrylic paint, in order to make the friction coefficient similar to that of human fingers on glass, and to increase brightness for imaging purposes. The artificial finger, and similar ones, have been previously shown to exhibit decreasing friction on a vibrating plate in proportion to the plate's amplitude [13], [7]. The same finger used here was also used in Wiertlewski et al; however, in the present study the inner foam layer has been replaced with a non-wetted foam to avoid introducing a source of humidity in the vacuum chamber [7].

### B. TPaD

Our ultrasonic friction reduction device, hereafter referred to as a TPaD (Tactile Pattern Display), is a glass plate 5x52x68 mm<sup>3</sup> in size. It is driven by 3 piezo actuators at 29075 Hz, a resonant mode of the glass at air pressure. A pickup piezo was placed along the same anti-nodal line as the finger in order to monitor vibration amplitude. The conversion between pickup piezo voltage and surface displacement at the point of finger contact was measured using a single point scanning vibrometer (Polytec, CLV 1000). The relationship was highly linear for all amplitudes used in these experiments.

TPaD vibration amplitude was kept consistent between trials using a PID control loop to adjust the actuation amplitude. Lowering the ambient air pressure from 1 atm to 0.017 atm results in a 17 Hz shift in the resonant frequency (see Fig. 1), and therefore in slightly different motion amplitude for a given input voltage amplitude. We chose to keep the actuating frequency constant and control only the input amplitude.

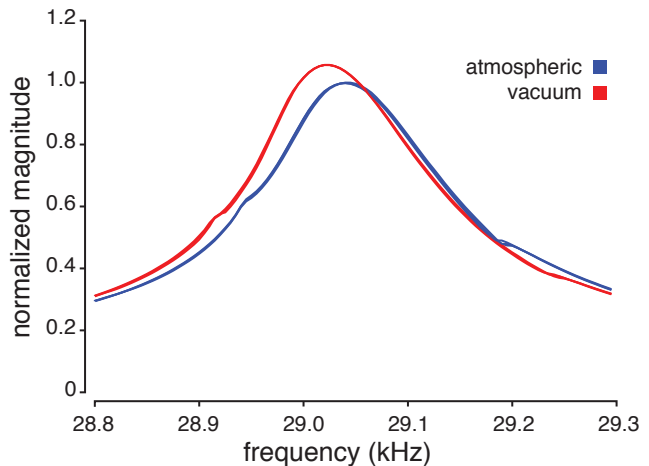


Fig. 1. Magnitude response of the TPaD for different input frequencies, in atmospheric and vacuum conditions. Three trials are shown for each condition; due to the TPaD's consistent response, same-condition trials lie closely atop one another. The removal of air causes the resonant frequency to shift by about 17 Hz and magnitude to increase by 5%. Measurements were taken at 5 Hz increments, which limits the acuity of exact peak location.

### C. Vacuum Chamber

The apparatus, shown in Fig. 2, consists of a single stage vacuum pump (model VP 135) attached to a 30cm wide domed vacuum chamber. A separate output tube on the opposite end of the chamber led to a piezo pressure sensor (Digi-Vac M2L760). Electrical connections passed through 2 mini DIN connectors through the bottom of the chamber, which were sealed around the wires with epoxy. Inside the chamber, the finger was attached to a small brushed DC motor. The chassis of the motor was mounted on a force sensor (Futek LSM250) to measure normal force. The finger contact patch could be viewed through the chamber wall when illuminated via frustrated total internal reflection.

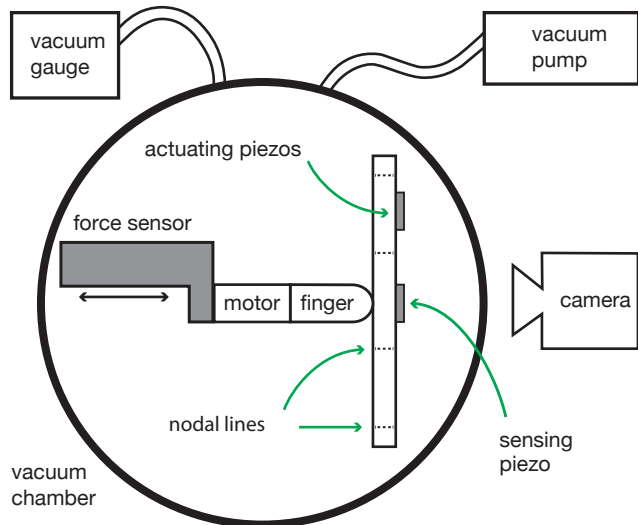


Fig. 2. Diagram of top down view of vacuum chamber apparatus. The finger rotates about its center axis perpendicular to the TPaD glass surface.

#### D. Motor Characterization

Traditionally, tribometric experiments are conducted by sliding the finger linearly to measure friction. Here, the finger rotated against the surface of glass; this still results in sliding motion between the finger contact patch and the glass, but without needing extra room for translation within the very limited space inside the vacuum chamber. Then any decrease in friction results in a higher torque on the motor rotating the finger, causing an observable increase in motor current and decrease in rotation speed.

A 5 volt DC brushed motor was used for all experiments. Motor current was measured across a 15 m $\Omega$  resistor. When freely spinning with the finger attached, the angular velocity of the motor  $\omega_m$  was 289 rad/s and the current was 35 mA. When the finger was stuck by higher frictional forces between the finger and TPaD, the motor stalled at 50 mA. For much of the data presented below, during TPaD activation the motor was neither stalled or free spinning, but rotated at an intermediate velocity. Terminal resistance can be calculated from  $R = V/i_{stall} = 100\Omega$ , resulting in a torque constant  $k_t = (V - Ri_{free})/\omega_m = 5.2 \times 10^{-3}$  Nm/A.

### III. DETECTING DIFFERENCES IN SLIDING FRICTION

#### A. Data Collection

For all experiments, “vacuum” refers to ambient pressure maintained at  $0.017 \pm 0.003$  atm, the lowest pressure possible with our pump and chamber. Before each experimental run, the finger and motor were slid forward into contact with the glass until a desired normal force was achieved. Following normal force adjustment, each experiment consisted of 6 data sets, 3 taken at atmospheric pressure and 3 at vacuum. Within a single data set, TPaD amplitude was varied sinusoidally at 0.25 Hz, allowing friction reduction to be observed over a wide range of amplitudes. All data is sampled at 50 Hz using an nScope data acquisition board developed at Northwestern University, and low-pass filtered in post processing to suppress electrical noise.

A representative experiment, consisting of 6 trials taken at 0.34 N normal force, is shown in Fig. 3. The top plot shows measured vibration amplitude, which is the envelope of the 29.075 kHz TPaD vibration, and is virtually identical between atmospheric and vacuum trials. Similarly, the normal force of the finger against the glass, shown in the bottom plot, remains consistent between trials. We monitored the normal force carefully out of concern that swelling of the finger in vacuum might affect it.

Friction was deduced by measuring the torque load on the spinning motor carrying the artificial finger. Higher friction on the finger corresponds to a higher torque, and thus higher measured motor current. Motor current is shown in the middle plot of Fig. 3.

#### B. Addressing Additional Sources of Error

In order to minimize the impact of trial order on results, atmospheric and vacuum trials were alternated; first an atmospheric pressure trial was collected, then a vacuum trial, then atmospheric again, etc. Every 6 trials, the finger was

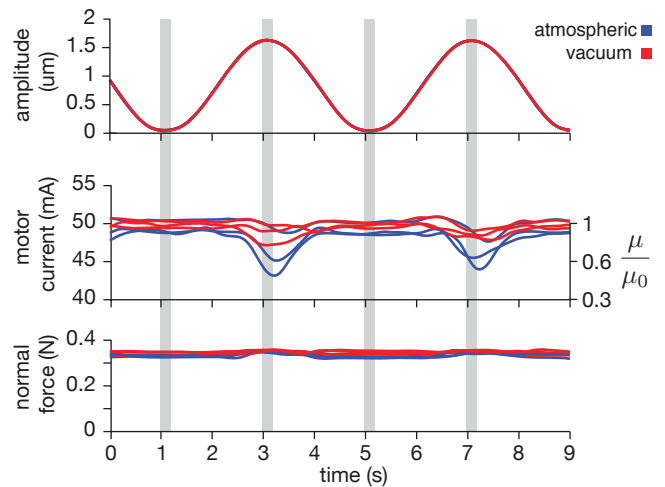


Fig. 3. A single experimental run consisting of 3 vacuum and 3 atmospheric trials, at 0.34 N finger normal force. Plots show, from top to bottom, (1) measured amplitude of TPaD vibration, (2) motor current representing finger friction, and (3) Normal force of the finger on the glass. Shaded regions show the windows where motor current data is extracted for comparison with other experiments in Fig. 5

removed and replaced, beginning a new experimental set. This ensured that experiments averaged out small variations in alignment and normal force.

Another concern was that the vibrating TPaD glass might heat up faster in vacuum than with air present. Higher temperature could affect frictional properties of the artificial finger. Therefore, an earlier version of the apparatus included a thermistor attached to the glass to monitor the temperature of the TPaD. We ascertained that the glass was not heating up significantly more in one pressure condition than the other.

#### C. Experimental Results

Six separate experiments, conducted at  $0.3 \pm 0.05$  N normal force, were collected over the course of 3 days. Additional experiments conducted outside of this normal force range were discarded due to the motor being mostly stalled at higher loads or always spinning with little load, which limited the range of torques the motor could experience. Differences in motor load between the different ambient pressures were readily apparent in much of the raw data; the central graph in Fig. 3 illustrates this clearly. The relationship between TPaD amplitude and motor current is more explicitly shown in Fig. 4. Here, a decrease in motor current, implying a decrease in friction on the TPaD surface, shows up at high amplitude for both atmospheric and vacuum trials, but the effect is noticeably greater at atmospheric pressure than in vacuum.

To compile results over multiple experiments, we compare the motor current between the minimum TPaD amplitude condition and the maximum amplitude condition. Current values were drawn from the shaded 0.2 second windows in Fig 3, when the TPaD was fully on or off. Then for each instance that the TPaD amplitude went from low to high, the difference between the starting and final current was found,

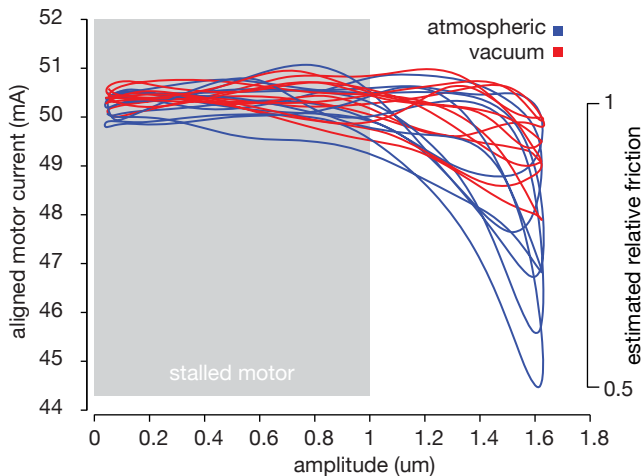


Fig. 4. Motor current plotted against TPaD amplitude for one 6-trial experiment conducted at 0.34 N. Data has been vertically aligned to have the same initial current at the start of each trial.

resulting in 6 current differences for each pressure condition for each experiment. The averages and standard error of these differences are shown in Fig. 5.

#### D. Model predictions

A partial squeeze film levitation model, described by equation 6, predicts that the effectiveness of the friction reduction is related to the ambient air pressure. The results of the simulation are shown Fig. 6 and composed of two steps. First, the average gap for any given vibration amplitude is numerically solved using the balance equation 5. Then, once the levitation is found, the squeeze number and the relative friction are calculated from their definitions, see equations 1 and 6. For the range of parameters used, the squeeze number remains above 150, which suggests that the behavior of the air trapped under the contact patch does not flow at the edges and its behavior is elastic.

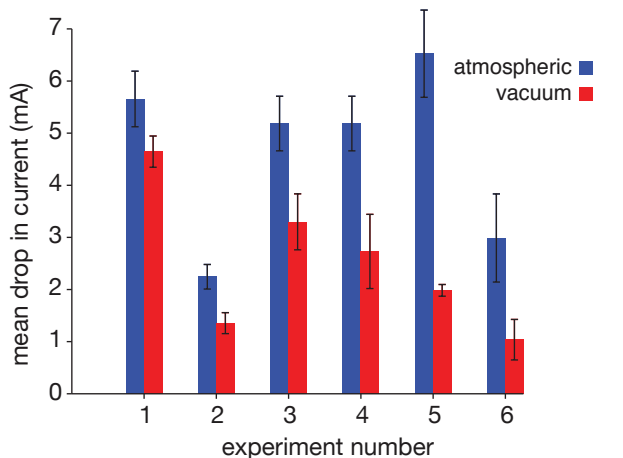


Fig. 5. Average decrease in motor current between maximum and minimum TPaD amplitudes, shown separately for atmospheric and vacuum trials. Each experiment was run at a normal force 0.35  $\pm$  0.05 N. Experiment 6 corresponds to data shown in Figs 3 and 4. Error bars are calculated from standard error of the mean of 6 runs for each pressure condition.

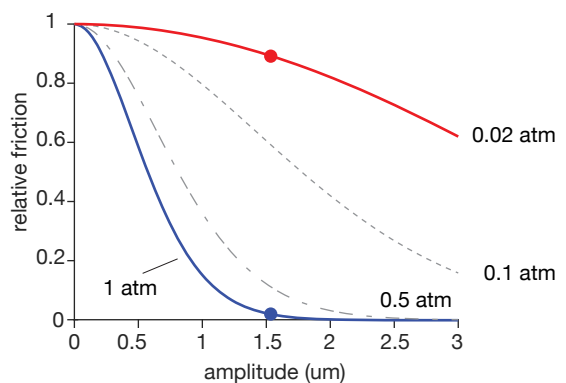


Fig. 6. Simulation results deriving from the partial levitation model for 100%, 50%, 10% and 2% of the ambient pressure. Estimation of the friction reduction curves. Maximum experimental amplitudes are indicated on their respective pressure curves. Under lower air pressure, more amplitude is needed to reach the same level of friction reduction.

The simulations take as input the roughness of the skin  $u_{rms} = 1.5 \mu\text{m}$ , the nominal gap  $u_0 = 4 \mu\text{m}$ , the contact length  $L = 10 \text{ mm}$  and the ambient pressure  $p_{atm}$  which varied from  $10^5 \text{ Pa}$  to  $0.02 \times 10^5 \text{ Pa}$ . The normal force was set to 0.3N to match the experimental conditions.

Simulation results show that for an amplitude of vibration of  $1.6 \mu\text{m}$ , the friction is reduced to 2% of its initial value under atmospheric pressure and to 88% of its initial value under at vacuum pressure (0.02 atm), a trend consistent with our experimental results. Discrepancies between predicted and actual amounts of friction reduction may be due to additional factors, such as bouncing. This is discussed further below.

#### IV. DISCUSSION

Our experiments show a change in friction reduction from atmospheric pressure conditions to that of near-vacuum. Of particular interest is a similarity between Figs. 4 and 6: at atmospheric pressure, the motor current (representing friction) decreases steeply at higher ultrasonic vibration amplitudes, while the drop off is much more modest at 0.02 atm. Importantly, in both the experiment and the model, the friction reduction effect does not entirely disappear in near vacuum.

Paired with previous work that shows fingers bouncing out of phase with the TPaD surface [8], this work suggests a nuanced description of how TPaDs might function. We know the finger is bouncing and in only intermittent contact (or near-contact), but we also now know that the amount of air in the system affects friction. The finger may essentially be bouncing on a film of air, with the presence of a squeeze film and the bouncing behavior (governed by material properties of the finger itself) both playing significant roles in friction reduction. The effects are most likely interdependent, and require further work to model them cohesively; note that the load-sharing squeeze film model in equation 6 shows friction levels depending on the size of the gap between surfaces, and this time averaged gap would depend on the phase and height

of bouncing.

Another interesting feature that has yet to be modeled is the lag in change of friction force when TPAD amplitude is changed. The delay can be seen as hysteresis in Fig. 4. It also appears across multiple tribometers; see [?] for another example. A delayed response to changes in TPAD amplitude could be due to a squeeze film taking some time to develop, perhaps as the dynamically bouncing finger settles into a new time-averaged gap distance.

Our method of measuring friction as presented in this paper has advantages: the work space is compact and can fit into a small vacuum chamber, motor current is a convenient indicator of friction force, and the finger is always in contact with the same region of the TPAD surface ensuring a consistent vibration amplitude. However, our particular motor only responded to changes in finger friction over a narrow range of normal forces, between 0.25 and 0.35 N, beyond which it was entirely stalled or else freely spinning. Future experiments, conducted at a variety of air pressures and over a wider range of TPAD amplitudes and finger contact normal forces, could expand the experimental evidence and inform improved models.

## V. CONCLUSION

In this paper, we presented evidence that friction reduction caused by ultrasonic vibration is affected by air pressure, confirming that squeeze film levitation is a likely contributor to friction reduction. Experimental results are broadly in line with our model predictions. We demonstrated a friction measurement method that can be used in a vacuum chamber.

## ACKNOWLEDGMENTS

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